QP CODE: 25050456

25050456

Reg No : .....

Name : .....

MSc DEGREE (CSS)EXAMINATION DECEMBER 2025

First Semester

M Sc MATHEMATICS

Core Courses - ME010104 - REAL ANALYSIS

2019 Admission Onwards

859757C5

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Let f be of bounded variation on $[a, b]$ and let V be defined as $V(x) = V_f(a, x)$, if $a < x \leq b$ and $V(0) = 0$. Then prove that every point of continuity of V is a point of continuity of f
2. Define rectifiable paths and arc lengths.
3. Define Riemann integral of a function f over $[a,b]$.
4. Give an example of a function f such that $f^2 \in \mathscr{R}$ *doesn't imply* $f \in \mathscr{R}$.
5. State the theorem of integration by parts.
6. Give an example to show that limit processes cannot be interchanged in general for sequence of functions.
7. Differentiate between pointwise convergence and uniform convergence of a sequence of functions.
8. State Cauchy criterion for uniform convergence of sequences of functions.
9. If $f_n(x) = \frac{x^2}{x^2+(1-nx)^2}$ $(0 \leq x \leq 1,\ n = 1, 2, 3, \dots)$, show that $\{f_n\}$ is not equicontinuous.
10. State Weierstrass approximation theorem.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Show that $f(x) = x^2 \cos \frac{1}{x}$ if $x \neq 0$ and $f(0) = 0$ is of bounded variation on $[0, 1]$.
12. Explain the terms orientation preserving functions and orientation reversing functions.
13. Prove that all continuous functions on $[a, b]$ are Riemann Stieltjes integrable.

14. *If $f \in \mathscr{R}(\alpha)$ and $g \in \mathscr{R}(\alpha)$ on [a,b], then show that $fg \in \mathscr{R}(\alpha)$ on [a,b] and $|f| \in \mathscr{R}(\alpha)$. Also show that $|\int_a^b f d\alpha| \leq \int_a^b |f| d\alpha$.*

15. Let α be monotonically increasing on $[a, b]$. Suppose $f_n \in \mathscr{R}(\alpha)$ on $[a, b]$, for $n = 1, 2, 3, \ldots$ and suppose $f_n \to f$ uniformly on $[a, b]$. Then prove that $f \in \mathscr{R}(\alpha)$ on $[a, b]$.

16. Obtain a series from $\phi(x) = |x|, (-1 \leq x \leq 1)$ and $\phi(x + 2) = \phi(x)$ for all real x, which converges uniformly on R^1.

17. Prove that a uniformly convergent sequence of continuous complex functions $\{f_n\}$ defined on a compact metric space K is equicontinuous.

18. For the double sequence $a_{ij}, i = 1, 2, 3, .., j = 1, 2, 3, \ldots$, suppose that $\sum_{j=1}^{\infty} |a_{ij}| = b_i$, $(i = 1, 2, 3, \ldots)$ and $\sum b_i$ converges. Prove that $\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} a_{ij}$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (i) Define total variation V_f of a real funcion f on $[a, b]$.
(ii) Prove that if f and g are of bounded variation on $[a, b]$, then so are their sum, difference and product. Also prove that
$V_{f \pm g} \leq V_f + V_g$ and $V_{f.g} \leq AV_f + BV_g$,
where $A = \sup\{|\, g(x) \,| : x \in [a, b]\}$ and $B = \sup\{|\, f(x) \,| : x \in [a, b]\}$.
(iii) State and prove additive property of total variation.

20. State and prove the theorem to change Riemann-Stieltjes integral to Riemann integral.

21. Suppose K is compact, and
(i) $\{f_n\}$ is a sequence of continuous functions on K,
(ii) $\{f_n\}$ converges pointwise to a continuous function f on K,
(iii) $f_n(x) \geq f_{n+1}(x), \forall x \in K, n = 1, 2, 3 \ldots$.
Then prove that $f_n \to f$ uniformly on K. Also show that if K is not compact, f_n does not converge to f uniformly.

22. Define exponential and logarithmic functions. Establish their properties and relationships between them.

(2×5=10 weightage)